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LIMITATION OF ACCELEROMETER TESTING

IN A

CONSTANT ONE GRAVITY FIELD

B

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## Introduction

The usual method of testing the response of an accelerometer is to orient the instrument so as to pick up varying components of the earth's gravitational field along the instrument's sensitive axis. This method is limited because a maximum of only one gravity unit of acceleration is available and also because the component of acceleration normal to the sensitive axis cannot be varied independently. However, with present techniques the precision obtainable far surpasses other methods such as the centrifuge, so that it is of interest to examine the above mentioned limitations in more detail. This memo is principally concerned with the consequences of the second limitation, namely, the interdependence of the accelerations parallel and normal to the sensitive axis.

The problem will be attacked by attempting to utilize the results of the one gravity test described above to determine the coefficients of a power series representation of the accelerometer performance. This representation is quite useful in system application of accelerometers and is appealing because the leading coefficients have a clear physical significance. It will be shown, however, that in general, this representation cannot be uniquely established on the basis of tests in a constant gravitational field alone.

## The Power Series Representation

Suppose the response of an accelerometer is to be represented by

$$A = \sum_{ij = 0}^{i + j = n} c_{ij} a_x^i a_y^j \qquad (1)$$

where $a_x$ and $a_y$ are the components of input acceleration along orthogonal x and y axes fixed in the accelerometer case and $c_{ij}$ are constant coefficients independent of $a_x$, $a_y$ and time. n will be called the degree of the series. This analysis is two-dimensional and thus is applicable only when the total acceleration lies in the x-y plane. However, it may also be applicable to physical three-dimensional cases in which the accelerometer is symmetric about one axis and can therefore be described in terms of just two inputs, namely the components of acceleration parallel and perpendicular to the axis of symmetry. The x-axis is termed the "sensitive axis" and the y-axis the "cross axis".

In an actual accelerometer, there will be uncertainties associated with each coefficient $c_{ij}$ due to measurement inaccuracies and variations with time, temperature and other environmental conditions (exclusive of steady accelerations). The $c_{ij}$'s are specifically defined to be independent of input accelerations, inasmuch as any such variation is already accounted for by the coefficients of higher degree terms. The analysis of the errors in the coefficients is not explicitly considered in this memo.

The power series is supposed to give a good representation over the entire dynamic range of the instrument and thus may sacrifice accuracy at one point (e.g., zero) to gain in accuracy over the rest of the range. Thus, we will not specifically consider effects peculiar to the zero point, such as threshold values and null shifts. Also, the assumed independence of time implicitly rules out consideration of effects dependent on the history of the unit such as hysteresis. For the purpose of this treatment, all such effects will be considered as errors or uncertainties.

The following nomenclature is suggested and will be used in discussion the various coefficients defined by equation (1):

$c_{00}$ = bias or zero off-set

$c_{10}$ = scale factor

$c_{20}$ = 2nd degree non-linearity

$c_{30}$ = 3rd degree non-linearity

$c_{01}$ = cross axis sensitivity

$c_{02}$ = 2nd degree cross axis sensitivity

$c_{11}$ = cross coupling coefficient

In a practical accelerometer, the scale factor should be much larger than the other coefficients, that is

$$c_{ij} \ll c_{10} \qquad \begin{matrix} i \neq 1 \\ j \neq 0 \end{matrix} \tag{2}$$

Cross Axis Sensitivity and Misalignment

It can easily be shown that a non-zero value of $c_{01}$ is completely equivalent to a misalignment of the "true" sensitive axis from that defined by the x-axis. We shall find an angle $\alpha$ defining a new set of axes x', y' such that $c'_{01}$ in the new system is zero. Let x', y' be such that

$$\begin{aligned} a_x &= a'_x \cos\alpha - a'_y \sin\alpha \\ a_y &= a'_x \sin\alpha + a'_y \cos\alpha \end{aligned} \tag{3}$$

Then, expanding equation (1), using (3) and rearranging terms, one can determine a new set of coefficients $c'_{ij}$ for the series in $a'_x$ and $a'_y$. In particular,

$$c'_{01} = c_{01}\cos\alpha - c_{10}\sin\alpha$$

so that if we choose

$$\alpha = \tan^{-1}(c_{01}/c_{10}) \tag{4}$$

$c'_{01}$ will vanish. In other words, one can always choose axes such that the cross axis sensitivity will be zero. Since we assume that $c_{01} \ll c_{10}$, the required rotation of axes will be small and the new axes will still be identified with the intuitive ideas of "sensitive" and "cross" axes

It may, of course, be possible to eliminate other coefficients by a suitable rotation of axes, but unless the rotation is small, it will grossly increase the values of the small coefficients, thus violating the relation (2) and thus our intuitive concept of the "sensitive" axis. For example, one cannot in general eliminate $c_{02}$ with small values of $\alpha$ since

$$c'_{02} = c_{02}\cos^2\alpha - c_{11}\sin\alpha\cos\alpha + c_{20}\sin^2\alpha$$

degree cross axis sensitivity ($c_{02}$) is of more fundamental importance than the linear term ($c_{01}$), the latter of which has no more or less significance than a simple axis misalignment.

Tests in a Constant One Gravity Field

In testing an accelerometer in the gravity field of the earth, the only input variable is the angle $\theta$ between the axes and the gravity vector. See Figure 1. (Since this is a two-dimensional analysis, we assume that the gravity vector lies in the x-y plane.)

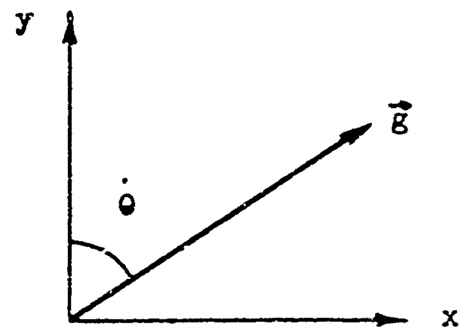


Figure 1

Therefore, the components of acceleration along x and y are

$$a_x = g \sin \theta \qquad\qquad (5)$$
$$a_y = g \cos \theta$$

Data from tests is known, however, only as a function of $\theta$ and not independently as a function of $a_x$ and $a_y$. In particular, the relation

$$a_x^2 + a_y^2 = g^2 \qquad\qquad (6)$$

is always true and herein lies the basic limitation of testing in a constant acceleration field. For example, we may now select an arbitrary constant $\lambda'$ to be added and subtracted to the right side of equation (1). By virtue of (6), we may express this constant as

$$\lambda' = \lambda g^2 = \lambda(a_x^2 + a_y^2)$$

and add to equation (1) the quantity

$$\lambda g^2 - \lambda(a_x^2 + a_y^2) = 0$$

This, however, results in a new set of coefficients $c''_{ij}$ which form an equally satisfactory description of the test data, namely,

$$c''_{00} = c_{00} + \lambda g^2$$
$$c''_{20} = c_{20} - \lambda \qquad\qquad (7)$$
$$c''_{02} = c_{02} - \lambda$$

with all other $c''_{ij} = c_{ij}$.

where $\lambda$ now is arbitrary. Without additional information or additional assumptions, the coefficients $c_{00}$, $c_{20}$ and $c_{02}$ cannot be uniquely determined by measurements in a constant acceleration field. On purely analytical grounds, this points up the need for precision measurements in acceleration fields other than the 1 "g" field of the earth. One possibility might be to make measurements of the same instrument at points on the earth's surface where different values of g exist. However, until the measurement errors are reduced by at least one order of magnitude the sensitivity of this experiment would be inadequate to determine a value for $\lambda$. It may be noted that with present techniques, the coefficients $c_0$, $c_2$ and $c_5$ for the best accelerometers are about the same order of magnitude as the measurement uncertainty, i.e. about $2 \times 10^{-5}$g. Another suggestion is to measure the accelerometer output in a zero gravity field, for example during the free-flight of a long range ballistic missile. This measurement would determine $c_{00}$ directly but might be of little value because effects due to the severe environmental conditions of powered flight. In some accelerometers, the effects of small angular velocities of the missile would directly effect the measurement.

## The Fourier Series Representation

Data from lg tests are obtained as a set of values $A_\mu = f(\theta_\mu)$. Since the data is by definition periodic in $\theta$, it is most logically analyzed by means of harmonic functions, for example the Fourier series

$$\begin{aligned} A_f &= a_0 + a_1\cos\theta + a_2\cos 2\theta + \dots \\ &\quad + b_1\sin\theta + b_2\sin 2\theta + \dots \\ &= \sum_{k=0}^{n} a_k\cos k\theta + \sum_{k=1}^{n} b_k\sin k\theta \end{aligned} \tag{8}$$

One will obtain values for the Fourier coefficients by a least squares fitting of the data (which will include more points than the number of coefficients sought.) The series may be extended as far as statistically meaningful non-zero values for the coefficients can be obtained. This condition is reached when the coefficients become small compared to the rms value of the residuals

$$R_\mu = A_\mu(\theta_\mu) - A_f(\theta_\mu).$$

## Determination of the Power Series Coefficients

As mentioned in the introduction, it is desirable to represent the accelerometer response with the power series expansion (1) instead of the Fourier series, and it is therefore required to transform the Fourier coefficients into the power series coefficients $c_{ij}$. The relationship is not unique, as was implied by the above dic-cussion. This is also evident when one observes that there are only $(2n + 1)$ Fourier coefficients compared to $(n^2/2 + 3n/2 + 1)$ power series coefficients. The discrepancy $(n^2 - n)/2$ is in fact just equal to the number of cross coupling terms, and represents the number of additional conditions necessary to determine the power series from the Fourier coefficients. It is assumed here that all terms above the $n$'th degree are zero.

The equations (7) show, however, that there is indeterminancy only in certain terms. If the terms up to second degree are considered, $c_{10}$, $c_{01}$ and $c_{11}$ are uniquely determined but $c_{00}$, $c_{20}$ and $c_{02}$ are not. Thus, one cannot arbitrarily fix any constants; one must fix one and only one of the constants in the set $(c_{00}, c_{20}, c_{02})$. By an extension of these arguments it can be shown that when third degree terms are included, the following sets of coefficients are subject to one

additional constraint each:

$$\left.\begin{array}{c} (c_{00},\ c_{20},\ c_{02}) \\ (c_{10},\ c_{12},\ c_{30}) \\ (c_{01},\ c_{21},\ c_{03}) \end{array}\right\} \quad (9)$$

$c_{11}$ is the only uniquely determined coefficient.

We can write down the equations relating the $c_{ij}$ to the Fourier coefficients by substituting (5) into (1), expanding in multiple angle terms and comparing term by term with (8). The relations for series including only terms of degree less than or equal to n are as follows:

<u>n = 0</u>

$$c_{00} = a_0 \quad (10)$$

<u>n = 1</u>

$$\left.\begin{array}{r} c_{00} = a_0 \\ c_{01} = a_1 \\ c_{10} = b_1 \end{array}\right\} \quad (11)$$

<u>n = 2</u>

$$\left.\begin{array}{r} 2c_{00} + c_{02} + c_{20} = 2a_0 \\ c_{01} = a_1 \\ c_{02} - c_{20} = 2a_2 \\ c_{10} = b_1 \\ c_{11} = 2b_2 \end{array}\right\} \quad (12)$$

<u>n = 3</u>

$$\left.\begin{array}{r} 2c_{00} + c_{02} + c_{20} = 2a_0 \\ 4c_{01} + 3c_{03} + c_{21} = 4a_1 \\ c_{02} - c_{20} = 2a_2 \\ -c_{03} - c_{21} = 4a_3 \\ 4c_{10} + 3c_{30} + c_{12} = 4b_1 \\ c_{11} = 2b_2 \\ -c_{30} + c_{12} = 4b_3 \end{array}\right\} \quad (13)$$

<u>n = 4</u>

$$\left.\begin{aligned}
8c_{00} + 4c_{02} + 4c_{20} + 3c_{04} + c_{22} + 3c_{40} &= 8a_0 \\
4c_{01} + 3c_{03} + c_{21} &= 4a_1 \\
c_{02} - c_{20} + c_{04} - c_{40} &= 2a_2 \\
-c_{03} - c_{21} &= 4a_3 \\
c_{04} - c_{22} + c_{40} &= 8a_4 \\
4c_{10} + 3c_{30} + c_{12} &= 4b_1 \\
2c_{11} + c_{13} + c_{31} &= 4b_2 \\
-c_{30} + c_{12} &= 4b_3 \\
c_{13} - c_{31} &= 8b_4
\end{aligned}\right\} \quad (14)$$

It is apparent that the $c_{ij}$'s are uniquely defined in terms of the $a_k$ and $b_k$ only for n = 0 and n = 1. Additional conditions are required to determine the coefficients for $n \geq 2$ as previously indicated. It should also be noted that the ambiguities in the $c_{ij}$ are not resolved simply by including higher degree terms from the Fourier series since each higher term brings in higher order unknown $c_{ij}$'s.

It may be possible, in any given situation, to set some of the $c_{ij}$ equal to zero based on auxiliary theoretical information. Alternately, one must make arbitrary assumptions, appealing to intuition for their justification. Suppose, for example, that one wished to determine a value for $c_{30}$ using a constant one gravity test. This will require determining the $a_k$'s and $b_k$'s at least up to k = 3. We will then require $(n^2 - n)/2 = 3$ additional assumptions in order to uniquely relate the Fourier coefficients to the power series coefficients. These assumptions, however, are restricted by (9). One possible set of assumptions is

$$\left.\begin{aligned}
&(1) \quad c_{12} = 0 \\
&(2) \quad c_{21} = 0 \\
&(3) \quad c_{02} = c_{20}(c_{01}/c_{10})
\end{aligned}\right\} \quad (15)$$

Combining (13) and (15) we obtain

$$\left.\begin{aligned}
c_{00} &= a_0 + a_2\left(\frac{c_{10} + c_{01}}{c_{10} - c_{01}}\right) \simeq a_0 + a_2 \\
c_{10} &= b_1 + 3b_3 \\
c_{01} &= a_1 + 3a_3 \\
c_{11} &= 2b_2 \\
c_{20} &= -2a_2\left(\frac{c_{10}}{c_{10} - c_{01}}\right) \simeq -2a_2 \\
c_{02} &= -2a_2\left(\frac{c_{01}}{c_{10} - c_{01}}\right) \simeq -2a_2(c_{01}/c_{10}) \\
c_{30} &= -4b_3 \\
c_{03} &= -4a_3
\end{aligned}\right\} \quad (16)$$

## Conclusion

It has been shown that testing accelerometers in a constant one gravity field has certain basic limitations associated with the fact that the acceleration components $a_x$ and $a_y$ are related by

$$a_x^2 + a_y^2 = g^2.$$

This makes it impossible to uniquely determine the coefficients of a power series expansion for the accelerometer performance (equation 1) from the results of a constant gravity test alone. The ambiguity cannot be resolved by higher precision tests but requires instead, tests under an acceleration field of different magnitude.

## Acknowledgement

The author is happy to acknowledge several interesting discussions with O. H. Griffith relative to the subject of this memo. Dr. Griffith has essentially verified the results by considering the transformation between the power series (1) and a slightly different representation of the experimental data.

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